average radius as the particle size. Such an estimate will obviously be the more precise, the narrower the range of particle sizes.

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PERTURBATION PROPAGATION IN NONLINEAR TRANSPORT PROCESSES DESCRIBED BY A TURBULENT FILTRATION EQUATION

K. B. Pavlov, A. S. Romanov, and I. A. Fedotov

UDC 532.516

A parabolic quasilinear equation of the form

$$\frac{\partial u}{\partial t}-\frac{\partial}{\partial x}\left(\left|\frac{\partial u^k}{\partial x}\right|^{n-1}\frac{\partial u^k}{\partial x}\right)=0,\quad k,n>0,\quad kn>1 \tag{1}$$

describes different transport processes in the case of a power-law dependence of the transport coefficients on the transportable quantity u and its gradient $\partial u/\partial x$. In particular, for $n = 1$ Eq. (1) can be considered as a nonlinear heat conduction equation, for $k = 1$ as the momentum transport in a non-Newtonian dilatant fluid, and in the general case of $k, n \neq 1$, as a turbulent filtration equation [1-3]. The essential feature of the transport processes described by (1) is the presence of the line $x = x_f(t)$ delimiting the domain with $u(x, t) = 0$ and the domain of localization of perturbations with $u(x, t) > 0$ [4]. Regularities of the motion of the front $x = x_f(t)$ in the Cauchy problem for (1) are investigated in this paper.

We shall consider an initial distribution of the transportable quantity described by the bounded finite function

$$u_0(x)\begin{cases}>0 & \text{for}\quad |x|<|x_\Phi|,\\ =0 & \text{for}\quad |x|>|x_\Phi|,\end{cases}$$

that is symmetric with respect to x to be given at the initial time $t = 0$, and assume that the asymptotic representation of the function $u_0(x)$ as $x \to x_\Phi + 0$, $x_\Phi < 0$ has the form

$$u_0(x)\sim U_0(x-x_\Phi)^\omega,\ \omega\geqslant 0. \tag{2}$$

Then the law of front motion $x_f = x_f(t)$ should be found from the solution of the Cauchy

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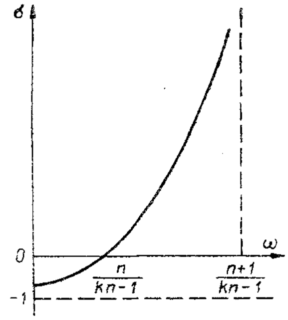

Fig. 1

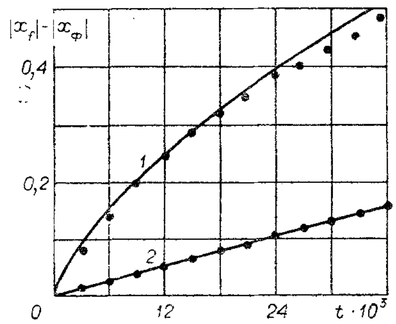

Fig. 2

problem

$$\frac{\partial u}{\partial t}-\frac{\partial}{\partial x}\left(\frac{\partial u^k}{\partial x}\right)^n=0,\quad 0>x>x_f(t),\quad t>0,$$
$$u(x,0)=u_0(x), \tag{3}$$

where it is necessary to consider $x_f(0) = x_\Phi$. It follows the condition of continuity of the desired solution $u(x, t)$ and of its derivatives $(\partial u^k/\partial x)^n$ on the front $x = x_f(t)$ that

$$u[x_f(t),t]=\left(\frac{\partial u^k}{\partial x}\right)^n[x_f(t),t]=0. \tag{4}$$

Differentiating the first of the conditions (4) along the line $x = x_f(t)$, we obtain the expression

$$\dot{x}_f\equiv\frac{dx_f}{dt}=-\lim_{x\to x_f+0}\frac{\partial u}{\partial t}\left(\frac{\partial u}{\partial x}\right)^{-1},$$

which can, by taking account of (3), be converted to the form

$$\dot{x}_f=-\lim_{x\to x_f+0}\frac{\partial}{\partial x}\left[\left(\frac{\partial u^k}{\partial x}\right)^n\right]\left(\frac{\partial u}{\partial x}\right)^{-1}. \tag{5}$$

from which the asymptotic representation for the transportable quantity follows

$$u(x,t)\sim\left[\frac{kn-1}{kn}\right]^{\frac{n}{kn-1}}(-\dot{x}_f)^{\frac{1}{kn-1}}(x-x_f)^{\frac{n}{kn-1}},\quad x\to x_f+0. \tag{6}$$

Equations (5) and (6) do not hold if $\dot{x}_f \equiv 0$.

Now, let us determine motion of the front as $t \to +0$. We shall start from the natural condition of the continuous passage of the solution $u = u(x, t)$ in the initial condition $u(x, 0) = u_0(x)$ as $t \to +0$

$$u(x,t)\sim U_0(x-x_\Phi)^\omega,\ t\to+0,\ x\to x_\Phi+0. \tag{7}$$

Taking into account that $(x - x_f)^\omega \sim (x - x_\Phi)^\omega$ as $t \to +0$, $x \to x_\Phi + 0$ and $x_f - x_\Phi = \int_0^t \dot{x}_f dt$, we obtain the following relationship from (6) and (7):

$$(-\dot{x}_f)^{\frac{1}{kn-1}}\left[-\int_0^t\dot{x}_f dt\right]^{\frac{n}{kn-1}-\omega}\sim U_0\left[\frac{kn}{kn-1}\right]^{\frac{n}{kn-1}},\quad t\to+0,\quad x\to x_\Phi+0.$$

We shall seek the function $\dot{x}_f = \dot{x}_f(t)$ in the form

$$\dot{x}_f=-At^\sigma<0, \tag{8}$$

then $A=\left\{\left(\frac{kn}{kn-1}\right)^n\beta^{\beta-1}U_0^{kn-1}\right\}^{1/\beta}$, $\sigma=-1+1/\beta, \beta=1+n+\omega-kn\omega$.

The dependence of the exponent $\sigma = \sigma(\omega)$ is shown in Fig. 1. For $0 < \omega < n/(kn-1)$ the velocity of front motion is $\dot{x}_f \to -\infty$ as $t \to +0$, but the transportable quantity remains localized since $|x_f - x_\Phi| < \infty$, because $\sigma > -1$, $0 < A < \infty$. If $\omega = n/(kn-1)$, then $\dot{x}_f(0) = [kn/(kn-1)]^n U_0^{kn-1}$. If $n/(kn-1) < \omega < (n+1)/(kn-1)$, then $\dot{x}_f \to 0$ as $t \to +0$.

It is interesting to consider the passage to the limit as $\omega \to (n+1)/(kn-1)$. In this case the derivatives are $d^jx_f/dt^j(0) = 0$ to $j = [\sigma] \to \infty$, hence in the limit it is necessary to consider $\dot{x}_f \equiv 0$ as $t \to +0$, and as noted, the expressions (5) and (6) become illegitimate.

Let us seek the asymptotic representation of the function $u = u(x, t)$ as $x \to x_f + 0$ directly in the form

$$u(x,\ t) \sim a(t)(x-x_\Phi)^\alpha,\ a(t)>0. \tag{9}$$

We then obtain from (3)

$$\frac{da}{dt} \sim a^{kn} n\,(k\alpha)^n\,(k\alpha-1)\,(x-x_\Phi)^{n(k\alpha-1)-1-\alpha},\quad x\to x_\Phi+0,$$

from which we determine

$$\alpha=\frac{n+1}{kn-1},\quad a=\left[\left(\frac{kn-1}{kn+k}\right)^n\frac{1}{n\,(k+1)\,(T-t)}\right]^{1/(kn-1)}.$$

The constant of integration T is evaluated from the initial condition (3)

$$T=U_0^{1-kn}\left(\frac{kn-1}{kn+k}\right)^n\frac{1}{n\,(k+1)}.$$

The asymptotic representation obtained for the solution

$$u\,(x,t)\sim a\,(t)\,(x-x_\Phi)^{\frac{n+1}{kn-1}},\quad x\to x_\Phi+0 \tag{10}$$

is valid only in the finite time $0 < t < T$ and is called metastable (see [5, 6], for instance).

Let us consider the auxiliary Cauchy problem that differs from (3) by the domain of definition $\infty > x > x_f(t)$, $t > 0$. The function

$$u^*\,(x,t)=\left\{\left(\frac{kn-1}{kn+k}\right)^n\frac{(x-x_\Phi)^{n+1}}{n\,(k+1)\,(T^*-t)}\right\}^{1/(kn-1)}, \tag{11}$$
$$\infty > x > x_\Phi,\ t<T^*=\text{const}<\infty$$

is a solution of the auxiliary problem with the appropriate initial condition containing the constant T^*.

Because of the monotonic dependence of the solution of the Cauchy problem for (1) on the initial condition, the function (11) majorizes any solution of the symmetric problem (3) [4] for a valid selection of the constant T^* for any $\omega \geqslant (n+1)/(kn-1)$. Therefore, for $\omega \geqslant (n+1)/(kn-1)$, a mode $x_f \equiv x_\Phi$ holds during at least the finite time segment $0 \leqslant t \leqslant T^*$.

To confirm the asymptotic relationship (8) obtained, the transport process described by (1) was computed numerically for $k = 1$, $n = 2$ with the initial condition

$$u\,(x,0)=\begin{cases}(1-|x|)^\omega, & |x|\leqslant 1,\\ 0, & |x|\geqslant 1.\end{cases}$$

The Cauchy problem formulated was supplemented by the boundary conditions $u(l, t) = 0$, $|l| = 2$, in the computations, as is possible if $|x_f(t)| < 2$. After quasilinearization near

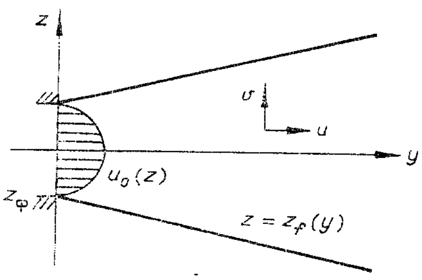

Fig. 3

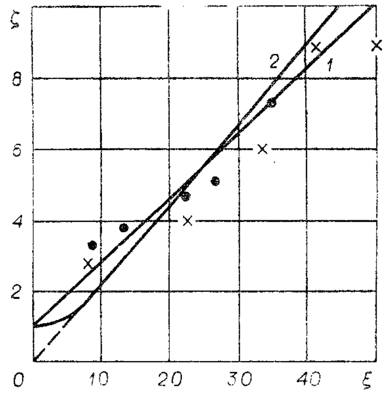

Fig. 4

the solution, the differential expression (1) was approximated by an implicit difference scheme of second order accuracy. The computations were performed by the factorization method. The location of the front $|x| = |x_f(t)|$ was determined approximately by the condition $u(x_f, t) = 10^{-5}u(0, t)$. Sufficient accuracy is obtained if the spacing $\Delta x = 0.02$, $\Delta t = 10^{-3}/3$ is selected in x and t, respectively.

Certain results of comparing the theoretical and numerical dependences are presented in Fig. 2, where motion of the localization domain front $|x_f| - |x_\Phi|$ with time is shown. Curves 1 and 2 are computed by means of (8) and correspond to values of the exponent $\omega = 1.5$; 2.0. The front location obtained numerically is indicated by the dots. For $\omega \geqslant 3$ metastable localization of the solution is set theoretically. For numerical computations the front remains fixed throughout the whole computation time to $t = 20\Delta t$ in this case. The computations performed in the range of variation $0 \leqslant \omega < 3$ of the exponent confirmed the relationship (8) completely.

As an application, let us use the theory developed to analyze the problem of a submerged turbulent jet of incompressible fluid of finite width (Fig. 3). The turbulent momentum transport is described in the boundary-layer theory approximation by the system of equations

$$u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial z} = \frac{1}{z^i}\frac{\partial}{\partial z}\left[z^i\frac{\tau}{\rho}\right],$$
$$\frac{\partial uz^i}{\partial y} + \frac{\partial vz^i}{\partial z} = 0, \quad \tau = \rho l_T^2\left|\frac{\partial u}{\partial z}\right|\frac{\partial u}{\partial z}, \tag{12}$$

where $i = 0.1$ corresponds to plane and cylindrical symmetry of the problem, ρ is the fluid density, τ is the turbulent friction stress determined by Prandtl [7, 8], and l_T is the turbulent mixing length. In the case of the submerged jet under consideration $l_T = cy$ [7, 8], where c is an empirical constant of the theory.

In the plane section of the nozzle y = 0 (Fig. 3), the fluid velocity equals

$$u(0, z) = u_0(z)\begin{cases} > 0 & \text{for } |z| < |z_\Phi|, \\ = 0 & \text{for } |z| > |z_\Phi|, \end{cases} \tag{13}$$

where the asymptotic representation for $u_0(z)$ as $|z| \to |z_\Phi| - 0$, $z_\Phi < 0$ will be considered given in the form

$$u_0(z) \sim W(|z_\Phi| - |z|)^\gamma, \; |z| \to |z_\Phi| - 0, \; W = \text{const} > 0, \; \gamma = \text{const} \geqslant 0. \tag{14}$$

A feature of a turbulent submerged jet in this formulation is its finite width. In other words, in any jet section $y = y_0$ there exists the jet boundary $z = z_f(y_0)$, $z_f(0) = z_\Phi$ such that $u(y_0, |z| < |z_f(y_0)|) > 0$ and $u(y_0, |z| > |z_f(y_0)|) = 0$. Physically obvious conditions of no velocity $u(y, \pm z_f) = 0$ and no turbulent friction stress $\partial u/\partial z(y, \pm z_f) = 0$ are satisfied on the jet boundary.

If we go over to new independent variables that are a generalization of the Mises variables [8] y, z → t, x, where

$$dt = \frac{1}{4} c^2 y^2 \left[(i + 1) \int\limits_0^{x_f} \frac{dx}{u} \right]^{3i/(i+1)} dy, \quad dx = uz^i dz, \tag{15}$$

then the initial problem (12)-(14) in the domain $0 > x > x_f$ will reduce to the Cauchy problem

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left\{ \left[1 + \int\limits_{x_f}^{x} \frac{dx}{u} \left(\int\limits_0^{x_f} \frac{dx}{u} \right)^{-1} \right]^{\frac{3i}{i+1}} \left(\frac{\partial u^2}{\partial x} \right)^2 \right\}, \quad x_f = x(y, z_f); \tag{16}$$

$$u(0, x) = u_0(x) \sim U_0(x - x_\Phi)^\omega, \quad x \to x_\Phi + 0, \; x_\Phi = x(0, z_\Phi); \tag{17}$$

$$u(t, x_f) = 0. \tag{18}$$

Here $U_0 = W\left[(Wz_\Phi^i)^{-1}(1 + \gamma)\right]^{\frac{\gamma}{1+\gamma}}; \quad \omega = \gamma/(1 + \gamma)$.

It follows from the condition of boundedness of the initial velocity distribution $0 \leqslant \gamma < \infty$ that $0 \leqslant \omega < 1$.

Limiting ourselves to the domain of the jet near the boundary $x \to x_f + 0$, we have

$$\int\limits_{x_f}^{x} \frac{dx}{u} \left(\int\limits_0^{x_f} \frac{dx}{u} \right)^{-1} \to 0,$$

hence, (16) is reduced to the form

$$\frac{\partial u}{\partial t} \sim \frac{\partial}{\partial x} \left\{ \left(\frac{\partial u^2}{\partial x} \right)^2 \right\}, \quad x \to x_f(t) + 0, \tag{19}$$

which agrees, together with conditions (17) and (18), with the problem (3) for $k = n = 2$.

The reverse passage to the physical variables y, z is accomplished by starting from the relationships (15).

Summarizing, the expressions

$$z_f \sim z_\Phi + 6^{1/3}\varkappa(1 - \omega)y, \; y \to +0, \; \varkappa^3 = 2c^2 \tag{20}$$

for a plane jet ($i = 0$) and

$$z_f \sim 2^{1/2}\left[z_\Phi^2/2 + 12^{2/3}(1 - \omega)^{1/3} t^{1/3}\right]^{1/2}, \quad t \to +0 \tag{21}$$

for a cylindrical jet ($i = 1$) are obtained for the jet boundary in conformity with the theory developed. Here $t = t(y)$ is evaluated from the relation

$$\left[z_\Phi^2/2 + 12^{2/3}(1 - \omega)^{1/3} t^{1/3}\right]^{-3/2} dt = 2^{3/2} c^2 dy^3/12.$$

Hence, if the inequality $y \gg z_\Phi$ is satisfied, then (21) is approximated by the relationship

$$z_f \sim 3^{1/3} 2(1 - \omega)^{1/3} \varkappa y, \; y \to +0 \; (i = 1). \tag{22}$$

Presented in Fig. 4 is a comparison between the theoretical dependences obtained for the jet boundary $z_f = z_f(y)$ and experimental results [9, 10] in dimensionless coordinates $\zeta = z_f/z_\Phi$ and $\xi = y/z_\Phi$. Curves 1 and 2 have been constructed by means of (20) and (21). Here we set $\omega = 0$, $\varkappa = 0.1$ and 0.077 for $i = 0$ and 1, respectively [7]. The crosses depict the experimental results for a plane turbulent jet ($i = 0$) [9], and the dots are for a cylindrical jet ($i = 1$) [10]. The dashed line has been constructed by means of the approximate dependence (22).

Agreement between the theoretical dependences obtained, the results of numerical computations, and the experimental data indicates the effectiveness of the asymptotic method developed.

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SPECTRAL STRUCTURE OF TURBULENT CONVECTION

I. V. Nikitina and A. G. Sazontov UDC 532.5

A central problem in the theory of evolution of strong turbulence is, as is well known, the determination of the spectrum of turbulence. Contemporary ideas on scale-invariant spectra are based on Kolmogorov's ideas, introducing the hypothesis of the self-similar nature of the spectrum in an inertial interval and the locality of turbulence [1]. For a long time similarity methods were essentially the only means of theoretical analysis for determining the spectral structure. However, due to the intermittent nature of turbulence dimensionality arguments often do not finally permit finding the form of the spectrum [2], therefore there have recently been numerous attempts at solving the problem of the Kolmogorov spectrum by starting directly from the equations of hydrodynamics.

The increasing interest in self-similar spectra is obviously related to two circumstances. First, the theory of scale-invariant spectra in phase transition problems has been substantially developed lately. Thus, the renormalized-group approach and consideration of problems in arbitrary dimensionality have been powerful means of studying critical effects [3, 4]; these ideas have by now been successfully transferred to strong turbulence [5, 6]. Secondly, the method of conformal mappings [7, 8], first suggested in [9] (see also the review [10]) for finding exact power-law solutions in the theory of weak turbulence, is quite fruitful in solving problems of the Kolmogorov spectrum.

So far all results on the spectra of strong turbulence referred to the case of an isotropic medium.* In reality the effect of anisotropy, related, for example, to the action of gravity forces, is important. In the present paper we solve the problem of finding anisotropic spectra of turbulent convection (the exceptional direction is the vertical).

The effect of convection plays a large role in many physical processes. For example, convective effects underlie a whole variety of solar phenomena [12]; convection is one of the

*Within weak turbulence anisotropic spectra were discussed in [11].

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